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LETTER TO THE EDITOR

Fourier–Gauss transforms of the Al-Salam–Chihara polynomials

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Abstract. We discuss classical Fourier–Gauss transforms of a three-parameter family of the continuous Al-Salam–Chihara polynomials $p_n(x; a, b|q)$. It is shown that they are related to both the continuous big q-Hermite $p_n(x; a, 0|q)$ and the q-Hermite $p_n(x; 0, 0|q)$ polynomials.

In this letter we examine the Fourier–Gauss transformation properties of a family of the three-parameter Al-Salam–Chihara polynomials [1]

$$p_n(x; a, b|q) := a^{-n}\,(ab; q)_n\, {}_3\phi_2\left[\begin{matrix} q^{-n}, a\,\mathrm{e}^{\mathrm{i}\theta}, a\,\mathrm{e}^{-\mathrm{i}\theta} \\ ab, 0 \end{matrix}; q, q\right] \tag{1}$$

in the variable $x = \cos\theta$. Here $(a; q)_0 = 1$ and $(a; q)_n = \prod_{j=0}^{n-1}(1 - aq^j)$, $n = 1, 2, 3, \ldots$, is the q-shifted factorial with the convention $(a_1, \ldots, a_k; q)_n = \prod_{j=1}^{k}(a_j; q)_n$ and ${}_3\phi_2$ is the basic hypergeometric series. Throughout this letter we will employ the standard notations of q-special functions [2, 3]. The Al-Salam–Chihara polynomials (1) are a particular case of the more general family of Askey–Wilson polynomials $p_n(x; a, b, c, d|q)$ [4] with two vanishing parameters $c = d = 0$. They are symmetric with respect to the parameters a, b and

$$p_n(-x; a, b|q) = (-1)^n\, p_n(x; -a, -b|q). \tag{2}$$

The Al-Salam–Chihara polynomials (1) with vanishing parameters a and b correspond to the continuous q-Hermite polynomials

$$H_n(x|q) := p_n(x; 0, 0|q) \tag{3}$$

of Rogers [5, 6]. At the next level of the Al-Salam–Chihara family one of the two parameters a and b is vanishing. This special case defines the continuous big q-Hermite polynomials [3, 7, 8]

$$H_n(x; a|q) := p_n(x; a, 0|q). \tag{4}$$

The Fourier–Gauss transformation properties of the continuous q-Hermite polynomials (3) and big q-Hermite polynomials (4) have been studied in [9] and [10], respectively. Since

§ On leave from: Institute of Physics, Azerbaijan Academy of Sciences, H.Javid pros. 33, Baku 370143, Azerbaijan. E-mail address: natig@matcuer.unam.mx the Al-Salam–Chihara polynomials can be expressed in terms of either the q-Hermite, or the big q-Hermite polynomials, their Fourier–Gauss transformation properties are bound to be connected as well. The purpose of the present letter is to make use of this circumstance in order to obtain classical Fourier–Gauss transforms of the Al-Salam–Chihara polynomials (1).

We begin with the relation

$$p_n(x;a,b|q) = \sum_{k=0}^{n} q^{k(k-1)/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (-a)^k H_{n-k}(x;b|q) \tag{5}$$

between the Al-Salam–Chihara (1) and big q-Hermite (4) polynomials. The coefficients of $H_{n-k}(x;b|q)$ in expansion (5) are a particular case of the general formula for the connection coefficients of the Askey–Wilson polynomials, derived in [4] by using the orthogonality relation for them. But one can find these coefficients directly from definitions (1) and (4). Indeed, from (4) and (1) it follows that

$$H_n(x;a|q) := a^{-n} {}_3\phi_2 \left[\begin{matrix} q^{-n}, a\,\mathrm{e}^{\mathrm{i}\theta}, a^{-\mathrm{i}\theta} \\ 0, 0 \end{matrix} ; q, q \right] = a^{-n} \sum_{k=0}^{n} \frac{(q^{-n};q)_k}{(q;q)_k} q^k (a\,\mathrm{e}^{\mathrm{i}\theta}, a\,\mathrm{e}^{-\mathrm{i}\theta}; q)_k. \tag{6}$$

Since

$$\frac{(q^{-n};q)_k}{(q;q)_k} = (-1)^k q^{k(k-1)/2-nk} \begin{bmatrix} n \\ k \end{bmatrix}_q \tag{7}$$

where $\begin{bmatrix} n \\ k \end{bmatrix}_q$ is the q-binomial coefficient

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{(q;q)_n}{(q;q)_k (q;q)_{n-k}} = \begin{bmatrix} n \\ n-k \end{bmatrix}_q \tag{8}$$

one can represent (6) as

$$H_n(x;a|q) = a^{-n} \sum_{k=0}^{n} (-1)^k q^{k(k+1)/2-nk} \begin{bmatrix} n \\ k \end{bmatrix}_q (a\,\mathrm{e}^{\mathrm{i}\theta}, a\,\mathrm{e}^{-\mathrm{i}\theta}; q)_k. \tag{6$'$}$$

It is now easy to invert expansion (6′) by the aid of the orthogonality relation [11]

$$\sum_{k=0}^{m} (-1)^k q^{k(k-1)/2} \begin{bmatrix} m \\ k \end{bmatrix}_q \begin{bmatrix} m-k \\ n \end{bmatrix}_q = \delta_{mn} \tag{9}$$

for the q-binomial coefficients (8). The inverse expansion is thus of the form

$$(a\mathrm{e}^{\mathrm{i}\theta}, a\mathrm{e}^{-\mathrm{i}\theta}; q)_k = \sum_{j=0}^{k} q^{j(j-1)/2} \begin{bmatrix} k \\ j \end{bmatrix}_q (-a)^j H_j(x;a|q). \tag{10}$$

Substituting (10) into the right member of (1), one obtains

$$p_n(x;a,b|q) = \frac{(ab;q)_n}{a^n} \sum_{k=0}^{n} q^{k(k+1)/2} \frac{(q^{-n};q)_k}{(ab,q;q)_k} (-a)^k H_k(x;a|q) \times {}_2\phi_1(q^{k-n}, 0; abq^k; q, q). \tag{11}$$

The basic hypergeometric series ${}_2\phi_1$ in (11) represents a special case of the Chu–Vandermonde q-sum

$${}_2\phi_1(q^{-n}, b; c; q, q) = \frac{(c/b;q)_n}{(c;q)_n} b^n \tag{12}$$

with the vanishing parameter b, so that for $k \leqslant n$ it is equal to

$$ {}_2\phi_1(q^{k-n}, 0; abq^k; q, q) = q^{n(n-1)/2-k(k-1)/2} \frac{(ab; q)_k}{(ab; q)_n} (-ab)^{n-k}. \tag{13} $$

Reversing the order of summation with respect to k in (11) and making use of (13) establishes the required relation (5).

Observe that from (5) and the limit relation [3]

$$ \lim_{q\to 1} \kappa^{-n} H_n(\kappa s; 2\kappa a|q) = H_n(s-a) \tag{14} $$

where $H_n(s)$ are the ordinary Hermite polynomials, it follows that

$$ \lim_{q\to 1} \kappa^{-n} p_n(\kappa s; 2\kappa a, 2\kappa b|q) = H_n(s-a-b). \tag{15} $$

Further, the inverse expansion with respect to (5) is

$$ H_n(x; b|q) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q a^{n-k}\, p_k(x; a, b|q). \tag{5$'$} $$

One easily checks (5′) by employing the same orthogonality relation (9) for the q-binomial coefficients (8). When $b = 0$ the expansion (5′) reduces to that for the q-Hermite polynomials $H_n(x|q)$ in terms of the big q-Hermite polynomials $H_n(x; a|q)$ [10].

It is essential to note that one can use (5′) for simple derivations of various expansions for the q-exponential function

$$ \mathcal{E}_q(x; t) = e_{q^2}(qt^2) \sum_{n=0}^{\infty} \frac{q^{n^2/4}}{(q; q)_n} t^n H_n(x|q) \tag{16} $$

on the q-quadratic lattice [12–15]. We mention here two instances of such a usage of (5′). Substitute first the expansion (5′) with $b = 0$ into the right-hand side of (16) and interchange the order of summation with respect to the indices n and k. This gives

$$ \mathcal{E}_q(x; t) = e_{q^2}(qt^2) \sum_{n=0}^{\infty} \frac{q^{n^2/4}}{(q; q)_n} t^n \varepsilon_q(atq^{n/2}) H_n(x; a|q) \tag{17} $$

where Jackson's q-exponential function $e_q(z)$ [16] and the q-exponential function on the q-linear lattice $\varepsilon_q(z)$ [17] are defined as

$$ e_q(z) := \sum_{n=0}^{\infty} \frac{z^n}{(q; q)_n} \qquad \varepsilon_q(z) := \sum_{n=0}^{\infty} \frac{q^{n^2/4}}{(q; q)_n} z^n. \tag{18} $$

Recall that the q-exponential functions $e_q(z)$ and $\varepsilon_q(z)$ are interrelated by the classical Fourier–Gauss transform [18]. When $a = q^{m/2}$ the expansion (17) in terms of the continuous big q-Hermite polynomials reduces to one algebraically derived in [7] by using symmetry techniques (the parameter b in [7] is connected with t in (17) by $b = -2it$).

In a like manner, from (17) and (5′) we have

$$ \begin{aligned} \mathcal{E}_q(x; t) = e_{q^2}(qt^2) \sum_{n=0}^{\infty} & \frac{q^{n^2/4}}{(q; q)_n} t^n p_n(x; a, b|q) E_{q^2}(abt^2 q^{n+1}) \\ & \times \varepsilon_q(\tfrac{1}{2}(\sqrt{a/b} + \sqrt{b/a}); q^{n/2}\sqrt{ab}t) \end{aligned} \tag{19} $$

where the q-exponential function

$$ E_q(z) := \sum_{n=0}^{\infty} \frac{q^{n(n-1)/2}}{(q; q)_n} (-z)^n \tag{20} $$

is the reciprocal to $e_q(z)$. In the passage from (17) to (19) we have employed the convolution-type relation

$$\mathcal{E}_q(\cos\theta; t) = e_{q^2}(qt^2) \sum_{n=0}^{\infty} \frac{q^{n^2/4}}{(q;q)_n} (t\mathrm{e}^{-\mathrm{i}\theta})^n \varepsilon_q(q^{n/2} t \mathrm{e}^{\mathrm{i}\theta}) \tag{21}$$

between the q-exponential functions on the q-quadratic and q-linear lattices $\mathcal{E}_q(\cos\theta; t)$ and $\varepsilon_q(q^{n/2}t\mathrm{e}^{\mathrm{i}\theta})$, respectively. One easily derives (21) by substituting the explicit form

$$H_n(\cos\theta|q) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathrm{e}^{\mathrm{i}(n-2k)\theta} \tag{22}$$

of the q-Hermite polynomials $H_n(x|q)$ in the relation (16) and interchanging then the order of summation with respect to the indices n and k.

The particular case of (19) with $a = q^{\alpha/2}$ and $b = q^{\beta/2}$ represents a q-analogue for the Al-Salam–Chihara polynomials of the Fourier–Gegenbauer expansion of a plane wave in terms of the Jacobi polynomials [19].

We return now to the expansion (5). The classical Fourier–Gauss transform of the continuous big q-Hermite polynomials (4) is known to be of the two alternative forms [10]

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(\sin\kappa s; a|q)\,\mathrm{d}s = \mathrm{i}^n q^{n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{3k^2/4-(n+1)k/2} \begin{bmatrix} n \\ k \end{bmatrix}_q \times (\mathrm{i}a)^k h_{n-k}(\sinh\kappa r|q) \tag{23a}$$

$$= \mathrm{i}^n q^{n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q c_{k,n}(q)(-\mathrm{i}a)^k h_{n-k}(\sinh\kappa r; a|q) \tag{23b}$$

where the constant $c_{k,n}(q)$ is equal to

$$c_{k,n}(q) = \sum_{j=0}^{k} \frac{(q^{-k};q)_j}{(q;q)_j} q^{(n+j/2)j/2}. \tag{23c}$$

Here $h_n(x|q) = \mathrm{i}^{-n} H_n(\mathrm{i}x|q^{-1})$ and $h_n(x;a|q) = \mathrm{i}^{-n} H_n(\mathrm{i}x;a|q^{-1})$ are the continuous q^{-1}-Hermite [6] and big q^{-1}-Hermite [10, 20] polynomials, respectively. They are interrelated by [10]

$$h_n(x;a|q) := \mathrm{i}^{-n} H_n(\mathrm{i}x;a|q^{-1}) = q^{-n(n-1)/2} \sum_{k=0}^{n} q^{k(k-1)/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (\mathrm{i}a)^{n-k} h_k(x|q) \tag{24a}$$

$$h_m(x|q) = \sum_{n=0}^{m} q^{n(n-m)} \begin{bmatrix} m \\ n \end{bmatrix}_q (-\mathrm{i}a)^{m-n} h_n(x;a|q). \tag{24b}$$

Hence it remains only to multiply both sides of expansion (5) by the factor $(2\pi)^{-1/2}\exp(\mathrm{i}sr - s^2/2)$ and integrate them over the variable s within infinite limits by using (23a), or (23b). This gives the following classical Fourier–Gauss transforms for the Al-Salam–Chihara polynomials

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} p_n(\sin\kappa s; a, b|q)\,\mathrm{d}s = \mathrm{i}^n q^{n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{3k^2/4-(n+1)k/2} \begin{bmatrix} n \\ k \end{bmatrix}_q \times (\mathrm{i}a)^k s_k(b/a;q) h_{n-k}(\sinh\kappa r|q) \tag{25a}$$

$$= \mathrm{i}^n q^{n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q (-\mathrm{i}a)^k A_{k,n}(b/a;q) h_{n-k}(\sinh\kappa r; a|q). \tag{25b}$$

Here

$$s_n(z;q) = \sum_{k=0}^{n} \frac{(q^{-n};q)_k}{(q;q)_k} q^{k(k+1)/2}(-z)^k = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{k(k-n)} z^k \tag{26}$$

are the Stieltjes–Wigert polynomials [2, 21] and the constant $A_{k,n}(b/a;q)$ is given by

$$A_{k,n}(a;q) = \sum_{j=0}^{k} (-1)^j q^{3j^2/4-kj+(n-1)j/2} \begin{bmatrix} k \\ j \end{bmatrix}_q s_j(a;q). \tag{27}$$

It is plain that when $b = 0$ the Fourier–Gauss transforms (25*a*) and (25*b*) reduce to (23*a*) and (23*b*), respectively.

One more form of the Fourier–Gauss integral (25) follows from $(5')$. Indeed, after transforming the base q into q^{-1}, expansion $(5')$ becomes

$$h_n(x;b|q) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{k(k-n)}(-\mathrm{i}a)^{n-k} \tilde{p}_k(x;a,b|q) \tag{28}$$

where $\tilde{p}_n(x;a,b|q) := \mathrm{i}^{-n} p_n(ix;a,b|q^{-1})$ are the q^{-1}-polynomials of Al-Salam and Chihara [20]. Now substituting (28) into the right member of (25*b*) results in

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr-s^2/2} p_n(\sin\kappa s;a,b|q)\,\mathrm{d}s = \mathrm{i}^n q^{n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q B_{k,n}(a,b;q) \times \tilde{p}_{n-k}(\sinh\kappa r;a,b|q) \tag{25c}$$

where the constant

$$B_{k,n}(a,b;q) = (-\mathrm{i}a)^k \sum_{j=0}^{k} q^{3j^2/4-kj+(n-1)j/2} \begin{bmatrix} k \\ j \end{bmatrix}_q (-b/a)^j s_j(a/b;q) s_{k-j}(b/a;q) \tag{29}$$

is symmetric with respect to the parameters a and b because of the simple property $s_n(z^{-1};q) = z^n s_n(z;q)$, enjoyed by the Stieltjes–Wigert polynomials (26).

The Fourier–Gauss integrals (25*a*)–(25*c*) thus transform the continuous Al-Salam–Chihara polynomials (1) into linear combinations of:

(a) the continuous q^{-1}-Hermite polynomials $h_n(x|q)$, which are a special case of the same family (1), but with vanishing parameters a and b;

(b) the continuous big q^{-1}-Hermite polynomials $h_n(x;a|q)$, which correspond to the next level of (1) with one vanishing parameter b;

(c) the continuous q^{-1}-polynomials of Al-Salam and Chihara from the same level (that is, when both parameters a and b are non-vanishing).

In a manner similar to the derivation of (25), from the inverse expansion to (28) and Fourier–Gauss transforms (23) one obtains

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}sr-s^2/2} \tilde{p}_n(\sinh\kappa s;a,b|q)\,\mathrm{d}s = \mathrm{i}^{-n} q^{-n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k^2/4+(1-n)k/2} \begin{bmatrix} n \\ k \end{bmatrix}_q \times (-a)^k H_k(b/a;q) H_{n-k}(\sin\kappa r|q) \tag{30a}$$

$$= \mathrm{i}^{-n} q^{-n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q a^k A_{k,n}(b/a;q^{-1}) H_{n-k}(\sin\kappa r;a|q) \tag{30b}$$

$$= q^{-n^2/4} \mathrm{e}^{-r^2/2} \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathrm{i}^{k-n} B_{k,n}(a,b;q^{-1}) p_{n-k}(\sin\kappa r;a,b|q). \tag{30c}$$

Here

$$H_n(z;q) := {}_2\phi_0(q^{-n}, 0; q, zq^n) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q z^k = s_n(z; q^{-1}) \tag{31}$$

are the Rogers–Szegö polynomials [22–25], and

$$A_{k,n}(a;q^{-1}) = \sum_{j=0}^{k} (-1)^j q^{j^2/4+(1-n)j/2} \begin{bmatrix} k \\ j \end{bmatrix}_q H_j(a;q) \tag{32}$$

$$B_{k,n}(a,b;q^{-1}) = (-\mathrm{i}a)^k \sum_{j=0}^{k} q^{j^2/4+(1-n)j/2} \begin{bmatrix} k \\ j \end{bmatrix}_q (-b/a)^j H_j(a/b;q) H_{k-j}(b/a;q). \tag{33}$$

Observe that Fourier–Gauss integrals (25*a*)–(25*c*) and (30*a*)–(30*c*) are interrelated by a replacement of the base $q \to q^{-1}$ (i.e., $\kappa \to \mathrm{i}\kappa$).

We note in closing that once the Fourier–Gauss transforms (25) and (30) between the continuous Al-Salam–Chihara polynomials $p_n(x; a, b|q)$ with different values of the parameter q are established, it means that we already understand the corresponding transformation properties of the Askey–Wilson family $p_n(x; a, b, c, d|q)$ up to its second level (that is, with two vanishing parameters $c = d = 0$). Now it seems quite plausible that our analysis of the Al-Salam–Chihara polynomials may be extended to the whole Askey–Wilson hierarchy (i.e., when all four parameters a, b, c, d have non-zero values). Work on clarifying this possibility is in progress.

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